

# Stewart

10.2 pg 653 #1-27 odds 31, 33, 35, 39

①  $x = t - t^3$   
 $y = 2 - 5t$

$$\frac{dy}{dx} = \frac{-5}{1-3t^2} \checkmark$$

③  $\frac{dy}{dx} = \frac{2\sin t \cos t}{\ln t + 1}$  ✓

⑤  $x = t^2 + t$   $t = 0$   $x = t \ln t$   
 $y = t^2 - t$   $y = \sin^2 t$

$$\frac{dy}{dx} = \frac{2t-1}{2t+1} \quad \frac{dy}{dx} \Big|_{t=0} = \frac{-1}{1} = \textcircled{-1}$$

$x(0) = 0$   $y(0) = 0$   $y = -1x$  ✓

⑦  $x = e^{\sqrt{t}}$   $t = 1$   
 $y = t - \ln t^2$

$$\frac{dy}{dx} = \frac{1 - \frac{1}{t^2} \cdot 2t}{\frac{1}{2} t^{-1/2} \cdot e^{\sqrt{t}}} = \frac{1 - \frac{2}{t}}{\frac{e^{\sqrt{t}}}{2\sqrt{t}}} = \frac{t-2}{t} \cdot \frac{2\sqrt{t}}{e^{\sqrt{t}}}$$

$$\frac{dy}{dx} = \frac{2\sqrt{t}(t-2)}{t e^{\sqrt{t}}} \quad \frac{dy}{dx} \Big|_{t=1} = \frac{2(-1)}{e} = \textcircled{\frac{-2}{e}}$$

$x(1) = e$   $y(1) = 1$   $y - 1 = \frac{-2}{e}(x - e)$  ✓

⑨  $x = e^t$   $(1, 1)$   $y - 1 = -2(x - 1)$  ✓  
 $y = (t-1)^2$

$t = \ln x$   
 $y' = 2(\ln x - 1) \cdot \frac{1}{x}$

$y' = \frac{2(\ln x - 1)}{x}$

$\frac{dy}{dx} = \frac{2(t-1)}{e^t}$

$1 = e^t$

$\ln 1 = t$

$t = 0$

$y' = \frac{2(-1)}{1} = \textcircled{-2}$

$\frac{dy}{dx} \Big|_{t=0} = \frac{2(-1)}{1} = \textcircled{-2}$

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$

(13)  $x = t^4 - 1$      $x' = 4t^3$   
 $y = t - t^2$      $y' = 1 - 2t$

$\frac{dy}{dx} = \frac{1-2t}{4t^3}$  ✓     $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-2(4t^3) - 12t^2(1-2t)}{16t^6}$

$\frac{d^2y}{dx^2} = \frac{4t-3}{16t^7}$  ✓

$= \frac{-8t^3 - 12t^2 + 24t^3}{16t^6}$

$= \frac{16t^3 - 12t^2}{16t^6}$

$= \frac{4t^2(4t-3)}{16t^6}$

$\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{4t-3}{4t^4} \cdot \frac{1}{4t^3}$

(15)  $x = \sin \pi t$      $y = \cos \pi t$   
 $x' = \pi \cos \pi t$      $y' = -\pi \sin \pi t$

$\frac{dy}{dx} = \frac{-\sin \pi t}{\cos \pi t} = -\tan \pi t$  ✓     $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-\pi \cos \pi t (\cos \pi t) + \pi \sin \pi t \cdot \pi \sin \pi t}{\cos^2 \pi t}$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-\pi (\cos^2 \pi t + \sin^2 \pi t)}{\cos^2 \pi t} = \frac{-\pi}{\cos^2 \pi t}$

$\frac{d^2y}{dx^2} = \frac{-\pi}{\cos^2 \pi t} \cdot \frac{1}{\pi \cos \pi t}$

$\frac{d^2y}{dx^2} = \frac{-1}{\cos^3 \pi t} = -\sec^3 \pi t$  ✓

17)  $x = e^{-t}$      $y = te^{2t}$   
 $\frac{dx}{dt} = -e^{-t}$      $\frac{dy}{dt} = 1e^{2t} + 2e^{2t} \cdot t$

$$\frac{dy}{dx} = \frac{e^{2t} + 2te^{2t}}{-e^{-t}} = -e^t \cdot e^{2t}(1+2t)$$

$$\frac{dy}{dx} = -e^{3t}(1+2t)$$

$$\frac{d}{dt} \left( \frac{dy}{dx} \right) = -3e^{3t}(1+2t) - e^{3t}(2)$$

$$\frac{d^2y}{dx^2} = \frac{-e^{3t}(3+6t+2)}{-e^{-t}}$$

$$\frac{d^2y}{dx^2} = e^{4t}(6t+5)$$

19)  $x = t(t^2-3)$   
 $\frac{dx}{dt} = 1(t^2-3) + 2t(t)$

$y = 3(t^2-3)$

when  $t = \pm\sqrt{3}$   
 $x \neq y = 0$   
 $(0, 0)$

$$\frac{dx}{dt} = t^2 - 3 + 2t^2$$

$$\frac{dy}{dt} = 6t$$

$$\frac{dx}{dt} = 3t^2 - 3$$

$$\frac{dy}{dt} = 0$$

$$t = 0$$

t	x	y
$-\sqrt{3}$	0	0
-1	+2	-6
0	0	-9
1	-2	-6
$\sqrt{3}$	0	0

$$\frac{dx}{dt} = 0$$

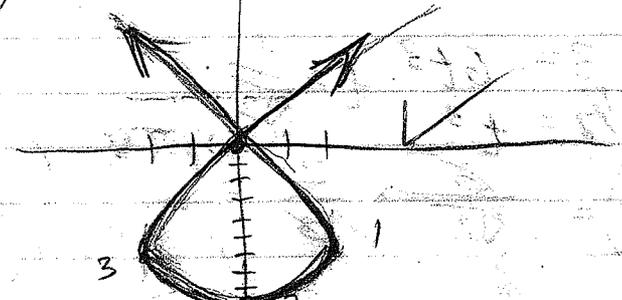
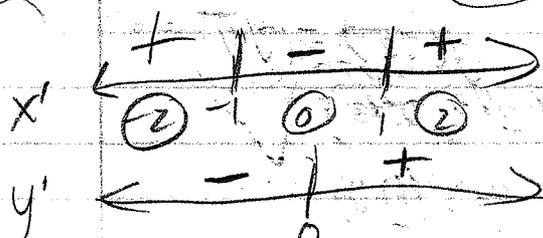
$$3t^2 - 3 = 0$$

$$t^2 = 1$$

$$t = \pm 1$$

Vertical  
 $x(1) = -2$   
 $y(1) = -6$   
 $x(-1) = +2$   
 $y(-1) = -6$

Horizontal  
 $x(0) = 0$   
 $y(0) = -9$



$$(11) \frac{dy}{dx} = \frac{2 \cos t}{4 \cos 2t} \quad (\sqrt{3}, 1) \quad 1 = 2 \sin t$$

$$\frac{dy}{dx} \Big|_{t=\pi/6} = \frac{2(\sqrt{3}/2)}{4(1/2)} = \frac{\sqrt{3}}{2}$$

$$\frac{1}{2} = \sin t$$

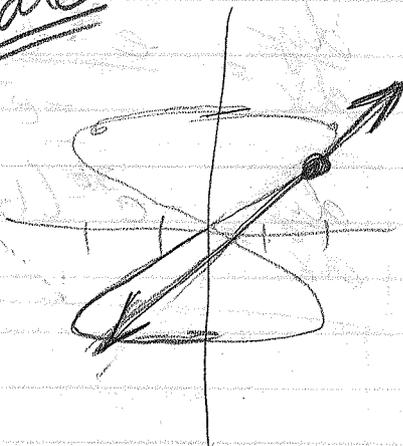
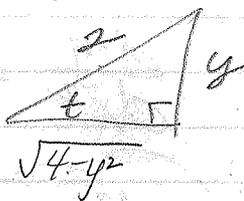
$$t = \pi/6$$

Calc

$$y - 1 = \frac{\sqrt{3}}{2}(x - \sqrt{3}) \quad \checkmark$$

$$x = 2 \cdot 2 \sin t \cos t \quad \frac{y}{2} = \sin t$$

$$x = 4 \cdot \frac{y}{2} \cdot \sqrt{4 - y^2}$$



$$x = 2y\sqrt{4 - y^2}$$

$$x = 2 \sin 2t$$

$$y = 2 \sin t$$

$$\frac{3(2^{-1/3})}{1+2^{-1}} = \frac{3(2)^{1/3}}{3/2}$$

$$y = \frac{3t^2}{1+t^3}$$

$$(21) \quad x = \frac{3t}{1+t^3}$$

$$\frac{3}{2^{1/3}} \cdot \frac{2}{3} = 2^{2/3}$$

$$\frac{dy}{dt} = \frac{6t(1+t^3) - 3t^2(3t^2)}{(1+t^3)^2}$$

$$\frac{dx}{dt} = \frac{3(1+t^3) - (3t^2)(3t)}{(1+t^3)^2}$$

$$\frac{dy}{dt} = \frac{6t + 6t^4 - 9t^4}{(1+t^3)^2}$$

$$\frac{dx}{dt} = \frac{3 + 3t^3 - 9t^3}{(1+t^3)^2}$$

$$\frac{dy}{dt} = \frac{-3t^4 + 6t}{(1+t^3)^2} = 0$$

$$\frac{dx}{dt} = \frac{-6t^3 + 3}{(1+t^3)^2} = 0$$

Horizontal

$$-3t(t^3 - 2) = 0$$

$$t = 0 \text{ and } \sqrt[3]{2}$$

Vertical  $3 = 6t^3$   $t = \frac{1}{\sqrt[3]{2}}$

$$\frac{1}{2} = t^3$$

$$x(0) = 0$$

$$y(0) = 0$$

$$x(\sqrt[3]{2}) = \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = 2^{1/3}$$

$$y(\sqrt[3]{2}) = \frac{\sqrt[3]{4}}{\sqrt[3]{2}} = 2^{2/3}$$

$$x\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{3}{\sqrt[3]{2}} \cdot \frac{2}{3} = 2^{2/3}$$

$\text{H} (0,0) (2^{1/3}, 2^{2/3})$   $\checkmark$

$$y\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{3}{3^{1/3}} \cdot \frac{2}{3} = \frac{2}{2^{1/3}} = 2^{2/3}$$

$\text{V} (2^{2/3}, 2^{1/3}) (0,0)$   $\checkmark$

27)  $x = \cos t$   
 $y = \sin t \cos t$

$(0, 0)$

$0 = \cos t$   
 $t = \pi/2, 3\pi/2, \dots$

$\frac{dy}{dx} = \frac{\cos t \cos t + -\sin t \sin t}{-\sin t}$

$0 = \sin t \cdot \cos t$   
 $t = \pi/2, \pi, 3\pi/2, 2\pi$

$\frac{dy}{dx} = \frac{\cos 2t}{-\sin t}$

when  $t = 0, \pi, 2\pi$

$\frac{dy}{dx} = \text{undefined}$

$y = \pm |x|$

on calc

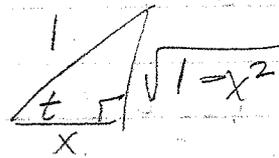
t	x	y
0	1	0
$\pi/2$	0	$1/2$
$\pi$	-1	0
$3\pi/2$	0	0
$2\pi$	1	0
$5\pi/2$	0	0
$3\pi$	-1	0
$7\pi/2$	0	0
$4\pi$	1	0

When  $t = \pi/2, 3\pi/2$

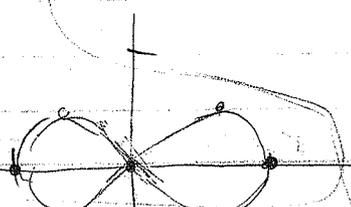
$\frac{dy}{dx} = \frac{-1}{-1} = (+1)$

$\frac{dy}{dx} = \frac{-1}{1} = (-1)$

$y = x\sqrt{1-x^2}$



x	y
0	0
$1/2$	$\sqrt{3}/4$



$\cos 2(\pi/2) = -1$   
 $-\sin \pi/2 = -1$

31)  $x = t^3 + 4t$   
 $y = 6t^2$

$x = -7t$        $t = \frac{x}{-7}$

$y = 12t - 5$   
 $y = 12(\frac{-x}{-7}) - 5$

$t = -1$  &  $-4/3$

$x(-1) = -5$        $x(-4/3) = -208/27$   
 $y(-1) = 6$        $y(-4/3) = 32/3$

$y = -\frac{12}{7}x - 5$

$(-5, 6)$  &  $(\frac{-208}{27}, \frac{32}{3})$

$\frac{12t}{3t^2+4} = -\frac{12}{7}$

$-3t^2 - 4 = 7t$

$0 = 3t^2 + 7t + 4$

$t^2 + 7t + 12$   
 $(t+3)(t+4)$

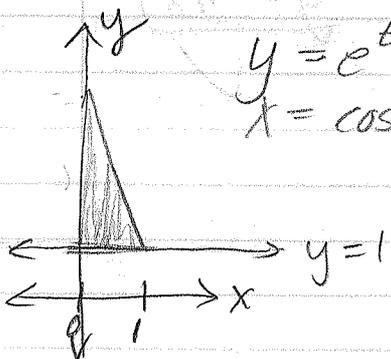
$(t+1)(3t+4) = 0$

(33)  $x=1$   
 $x=0$   $\int y-1 dx = \int_0^{\pi/2} (e^t - 1) \cdot (-\sin t) dt$

$1 = e^t$   $x(0) = 1$   
 $\ln 1 = t$   $y(0) = 1$

$t=0$

$\int_{\pi/2}^0 +\sin t - e^t \sin t dt \approx$   
 $+1.905 \checkmark$



$y = e^t$   $\ln y = t$   
 $x = \cos t$   $x = \cos(\ln y)$

$x = \cos t$

$0 = \cos t$   
 $t = \pi/2, 3\pi/2$

to get new endpoints

$1 = \cos t$   
 $t = 0, 2\pi$

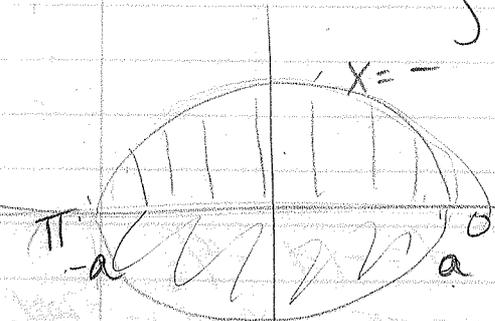
(35)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   $2 \int_{x=-a}^{x=a} y dx = -2 \int_{\theta=0}^{\theta=\pi} b \sin \theta \cdot -a \sin \theta d\theta$

$x = a \cos \theta$   
 $y = b \sin \theta$

$2 \int_{x=-a}^{x=a} y dx$

$+2ab \int_{\pi}^0 \sin^2 \theta d\theta$

$2ab \int \frac{1 - \cos 2\theta}{2} d\theta$



$ab \int 1 - \cos 2\theta d\theta =$

$ab \left[ \theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi} =$

$-a = a \cos \theta$   $a = a \cos \theta$

$-1 = \cos \theta$   $1 = \cos \theta$

$\theta = \pi$

$\theta = 0$

$ab \left[ \pi - \frac{1}{2} \sin(\pi) - 0 + \frac{1}{2} \sin(0) \right] = +\pi ab \checkmark$

(39)  $x = \sin t - 2 \cos t$   
 $y = 1 + \sin t \cos t$

$0 = \sin t - 2 \cos t$

$2 \cos t = \sin t$

$2 = \frac{\sin t}{\cos t}$

$2 = \tan t$

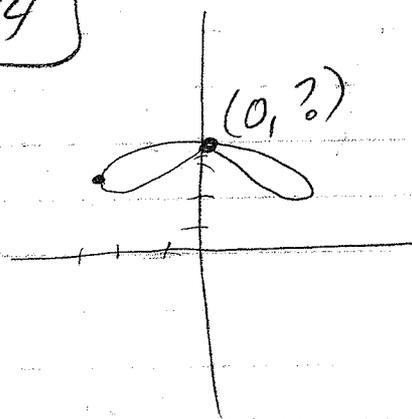
$t = 1.107, 4.249$

Let  $A = 1.107$

$B = 4.249$

$\int_A^B (1 + \sin t \cos t)(\cos t + 2 \sin t) dt$

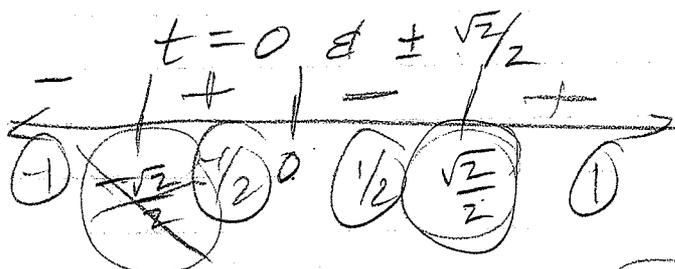
$\approx 0.8944$



(23)  $x = t^4 - t^2$   
 $x' = 4t^3 - 2t$   
 $x' = 2t(2t^2 - 1)$

$t \in (0, \infty)$   $y = t + \ln t$

$t^2 = 1/2$



$(-\frac{1}{4}, \frac{\sqrt{2}}{2} + \ln \frac{\sqrt{2}}{2})$

$\frac{1}{2} \ln 2 - \ln 2$

$x(\frac{\sqrt{2}}{2}) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$y(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} + \ln \frac{\sqrt{2}}{2}$

10.3

$$\textcircled{1} \int_0^2 \sqrt{1+4t^2} dt \quad \checkmark$$

$$\textcircled{2} \int_0^\pi \sqrt{\sin^2 t + 2t \sin t \cos t + t^2 \cos^2 t + \cos^2 t - 2t \cos t \sin t + t^2 \sin^2 t} dt$$

$$\sqrt{t^2(\sin^2 t + \cos^2 t) + \cos^2 t + \sin^2 t} dt$$

$$\int_0^\pi \sqrt{t^2 + 1} dt \quad \checkmark$$

10.3 pg. 659 #1, 3, 5, 9, 11, 21, 23, 25 & 29

①  $x = t - t^2$      $y = \frac{4}{3}t^{3/2}$      $1 \leq t \leq 2$   
 $\frac{dx}{dt} = 1 - 2t$      $\frac{dy}{dx} = 2t^{1/2}$

$$\int_1^2 \sqrt{(1-2t)^2 + (2t^{1/2})^2} dt = \int_1^2 \sqrt{(1-2t)^2 + 4t} dt$$

③  $x = t \sin t$      $y = t \cos t$      $0 \leq t \leq \pi/2$   
 $\frac{dx}{dt} = 1 \sin t + t \cos t$      $\frac{dy}{dt} = 1 \cos t - t \sin t$

$$\int_0^{\pi/2} \sqrt{(\sin t + t \cos t)^2 + (\cos t - t \sin t)^2} dt$$

⑤  $x = t^3$      $y = t^2$      $0 \leq t \leq 4$   
 $\frac{dx}{dt} = 3t^2$      $\frac{dy}{dt} = 2t$

$$\int_0^4 \sqrt{(3t^2)^2 + (2t)^2} dt = \int_0^4 \sqrt{9t^4 + 4t^2} dt$$

$$\int_0^4 \sqrt{t^2(9t^2 + 4)} dt = \int_0^4 t \sqrt{9t^2 + 4} dt =$$

$u = 9t^2 + 4$   
 $du = 18t dt$   
 $dt = \frac{du}{18t}$

$$\int_4^{148} \frac{t \sqrt{u} \cdot du}{18t} = \frac{1}{18} \int_4^{148} u^{1/2} du =$$

$$\frac{1}{18} \left[ \frac{2}{3} u^{3/2} \right]_4^{148} = \frac{1}{27} (148)^{3/2} - \frac{1}{27} (4)^{3/2}$$

$$\frac{148 \sqrt{148} - \frac{8}{27}}{27}$$

$$x = e^t \cos t$$

$$y = e^t \sin t$$

$$0 \leq t \leq \pi$$

$$\frac{dx}{dt} = -e^t \sin t + e^t \cos t$$

$$\frac{dy}{dt} = e^t \cos t + e^t \sin t$$

9

$$\int_0^\pi \sqrt{(-e^t \sin t + e^t \cos t)^2 + (e^t \cos t + e^t \sin t)^2} dt =$$



$$\int_0^\pi \sqrt{2} e^t dt = \sqrt{2}(e^\pi - 1) \approx 31.312$$

Arclength

11

$$x = e^t - t$$

$$\frac{dx}{dt} = e^t - 1$$

$$y = 4e^{t/2}$$

$$\frac{dy}{dt} = 2e^{t/2}$$

$$-8 \leq t \leq 3$$

$$\int_{-8}^3 \sqrt{(e^t - 1)^2 + (2e^{t/2})^2} dt = 31.085$$

$$e^3 - e^{-8} + 11$$

Arclength

21

$$x = t^3$$

$$y = t^4$$

$$0 \leq t \leq 1$$

$$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 4t^3$$

$$2\pi \int_0^1 t^4 \sqrt{(3t^2)^2 + (4t^3)^2} dt$$

SA  
x-axis

23

$$x = t^3$$

$$0 \leq t \leq 1$$

$$2\pi \int_0^1 t^2 \sqrt{9t^4 + 4t^2} dt$$

SA  
x-axis

$$\frac{dx}{dt} = 3t^2$$

$$y = t^2$$

$$\frac{dy}{dt} = 2t$$

$$2\pi \int_0^1 t^3 \sqrt{9t^2 + 4} dt$$

$$2\pi \int_0^1 t^3 \sqrt{9t^2 + 4} dt = \frac{2\pi}{1215} (247\sqrt{13} + 64)$$

SA X-axis

(25)  $x = a \cos^3 \theta$      $y = a \sin^3 \theta$      $0 \leq \theta \leq \pi/2$

$\frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta$      $\frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta$

$$2\pi \int_0^{\pi/2} a \sin^3 \theta \sqrt{(-3a \cos^2 \theta \sin \theta)^2 + (3a \sin^2 \theta \cos \theta)^2} d\theta$$

$$= 2\pi \int_0^{\pi/2} a \sin^3 \theta \sqrt{9a^2 \cos^4 \theta \sin^2 \theta + 9a^2 \sin^4 \theta \cos^2 \theta} d\theta$$

$$= 2\pi \int_0^{\pi/2} a \sin^3 \theta \cdot \sqrt{9a^2 \cos^2 \theta \sin^2 \theta (\cos^2 \theta + \sin^2 \theta)} d\theta$$

$$2\pi \int_0^{\pi/2} a \sin^3 \theta \cdot 3a \sin \theta \cos \theta d\theta$$

$u = \sin \theta$   
 $du = +\cos \theta d\theta$   
 $d\theta = \frac{du}{\cos \theta}$

$$6\pi a^2 \int_0^{\pi/2} \sin^4 \theta \cdot \cos \theta d\theta$$

$$= 6\pi a^2 \int_0^1 u^4 \cdot \cos \theta \cdot \frac{du}{\cos \theta}$$

$$+ 6\pi a^2 \int_0^1 u^4 du = +6\pi a^2 \left[ \frac{1}{5} u^5 \right]_0^1 =$$

$$+6\pi a^2 \cdot \left[ \frac{1}{5} - 0 \right] = \boxed{+\frac{6}{5}\pi a^2}$$

\* (29)  $x = 3t^2$      $y = 2t^3$      $0 \leq t \leq 5$

$\frac{dx}{dt} = 6t$      $\frac{dy}{dt} = 6t^2$

SA Y-axis

$$2\pi \int_0^5 3t^2 \sqrt{36t^2 + 36t^4} dt = 2\pi \int_0^5 3t^2 \cdot \sqrt{36t^2(1+t^2)} dt$$

$$= 6\pi \int_0^5 t^3 \cdot 6 \sqrt{1+t^2} dt$$

$u = 1+t^2$   
 $du = 2t dt$

$$18\pi \int_1^{26} t^2 \sqrt{u} \frac{du}{2t}$$

$$36\pi \int_0^5 t^3 \sqrt{1+t^2} dt$$

$$= 18\pi \int_1^{26} u^{3/2} - u^{1/2} du = 18\pi \left[ \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right]_1^{26}$$

(29) continued

$$18\pi \left[ \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right]_{1}^{26}$$

$$36\pi \left( u^{3/2} \left[ \frac{1}{5} u' - \frac{1}{3} \right] \right)_{1}^{26}$$

$$\frac{72\pi}{15} (959\sqrt{26} + 1)$$

$$36\pi \left[ 26\sqrt{26} \left[ \frac{26}{5} - \frac{1}{3} \right] - 1 \left[ \frac{1}{5} - \frac{1}{3} \right] \right]$$

$$\frac{24\pi}{5} (959\sqrt{26} + 1)$$

$$36\pi \left[ 26\sqrt{26} \left( \frac{78-5}{15} \right) - \left( \frac{3-5}{15} \right) \right]$$

$$\frac{72\pi}{15} (13\sqrt{26} \cdot 73 + 1)$$

$$36\pi \left[ 26\sqrt{26} \left( \frac{73}{15} \right) + \frac{2}{15} \right]$$

$$\begin{array}{r} \frac{1}{3} \\ \times 13 \\ \hline 229 \\ 730 \\ \hline 959 \end{array}$$

$$\frac{18\pi}{32} \left[ \frac{2}{5} (109)^{5/2} - 6 (109)^{3/2} \right] - \frac{18\pi}{32} \left[ \frac{486}{5} - 162 \right]$$

$$\frac{9\pi}{8} [ 11881\sqrt{109} - 327\sqrt{109} ]$$

$$\frac{9\pi}{8} [ 109\sqrt{109} + 118 ]$$

$$\frac{9\pi}{8} [ 109\sqrt{109} + 118 ]$$

$$\frac{24\pi}{5} (959\sqrt{26} + 1)$$

10.4A

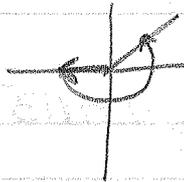
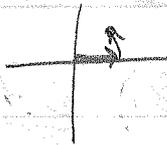
5) a) (1, 1)

i)  $(\sqrt{2}, \pi/4)$

ii)

$(-\sqrt{2}, 5\pi/4)$

$$\begin{aligned} 1^2 + 1^2 &= r^2 \\ 2 &= r^2 \\ r &= \sqrt{2} \end{aligned}$$



$$\tan \theta = 1$$

$$\theta = \pi/4$$

b)  $(2\sqrt{3}, -2)$

$$\tan \theta = \frac{-2}{2\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\begin{aligned} (2\sqrt{3})^2 + (-2)^2 &= r^2 \\ 12 + 4 &= r^2 \\ 16 &= r^2 \\ r &= 4 \end{aligned}$$

$$\theta = \frac{11\pi}{6}$$

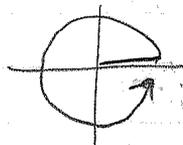
Q4

i)

$(4, 11\pi/6)$

ii)

$(-4, 5\pi/6)$



13)  $(1, \pi/6)$  &  $(3, 3\pi/4)$

$$\begin{aligned} x &= 1 \cos \pi/6 \\ x &= \sqrt{3}/2 \\ y &= 1 \sin \pi/6 \\ y &= 1/2 \end{aligned}$$

$$\begin{aligned} x &= 3 \cos 3\pi/4 \\ x &= 3 \cdot \frac{-\sqrt{2}}{2} \\ x &= \frac{-3\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} y &= 3 \sin 3\pi/4 \\ y &= \frac{3\sqrt{2}}{2} \end{aligned}$$

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\left(\frac{-3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$$

$$D = \sqrt{\left(\frac{\sqrt{3}}{2} + \frac{3\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2} - \frac{3\sqrt{2}}{2}\right)^2}$$

$$D = \sqrt{\left(\frac{3\sqrt{2} + \sqrt{3}}{2}\right)^2 + \left(\frac{1 - 3\sqrt{2}}{2}\right)^2}$$

$$D = \sqrt{\frac{18 + 6\sqrt{6} + 3 + 1 - 6\sqrt{2} + 18}{4}}$$

$$D = \sqrt{\frac{40 + 6\sqrt{6} - 6\sqrt{2}}{2}}$$

$$(15) r=2$$

$$\boxed{x^2 + y^2 = 4} \checkmark$$

$$(17) r = 3 \sin \theta$$

$$r^2 = 3r \sin \theta$$
$$x^2 + y^2 = 3y$$

$$x^2 + y^2 - 3y + \frac{9}{4} = 0 + \frac{9}{4}$$

$$\boxed{x^2 + (y - \frac{3}{2})^2 = \frac{9}{4}} \checkmark$$

$$(23) x^2 + y^2 = 25$$

$$\boxed{r=5} \checkmark$$

$$(29) r = -2 \sin \theta$$

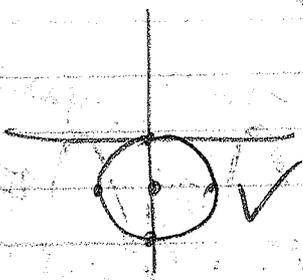
$$r^2 = -2r \sin \theta$$

$$x^2 + y^2 = -2y$$

$$x^2 + y^2 + 2y + 1 = 0 + 1$$

$$\boxed{x^2 + (y+1)^2 = 1} \checkmark$$

$$(0, -1) \quad r=1$$



$$(19) r^2 = \sin 2\theta$$

$$r^2 = 2 \sin \theta \cos \theta$$

$$r^4 = 2r \sin \theta \cdot r \cos \theta$$

$$\boxed{(x^2 + y^2)^2 = 2yx} \checkmark$$

$$(21) y=5$$

$$r \sin \theta = 5$$

$$\boxed{r = 5/\sin \theta \text{ OR } 5 \csc \theta} \checkmark$$

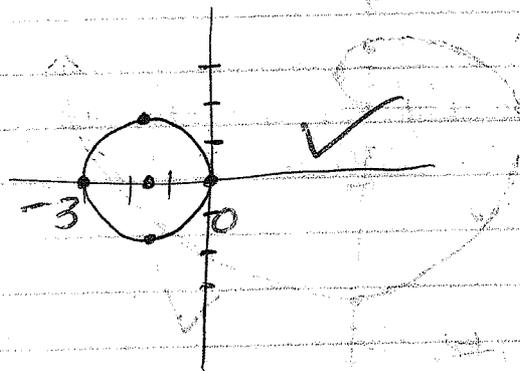
$$(25) 2xy = 1$$

$$2r \cos \theta \cdot r \sin \theta = 1$$

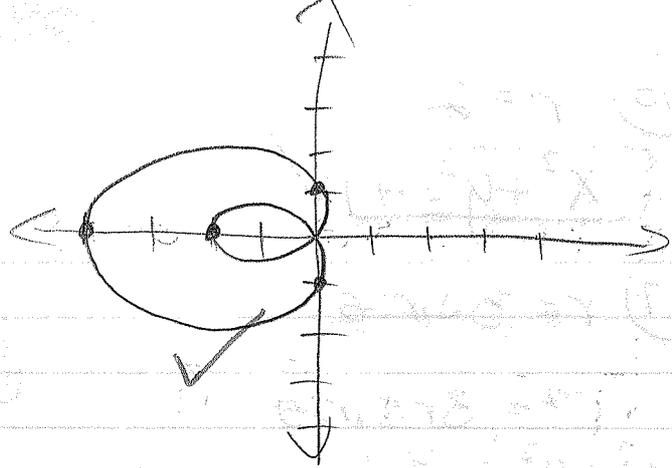
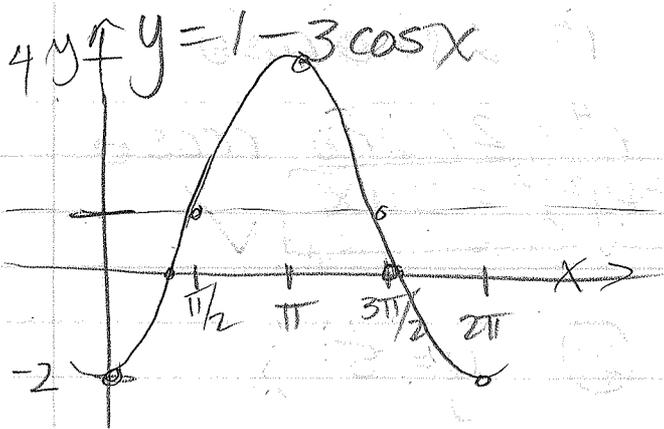
$$r^2 = \frac{1}{2 \sin \theta \cos \theta} = \frac{1}{\sin 2\theta}$$

$$\boxed{r^2 = \csc 2\theta} \checkmark$$

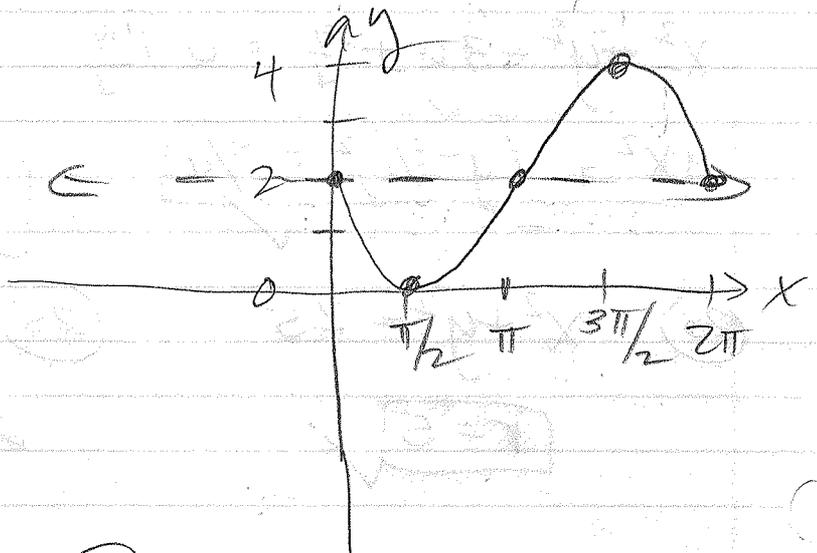
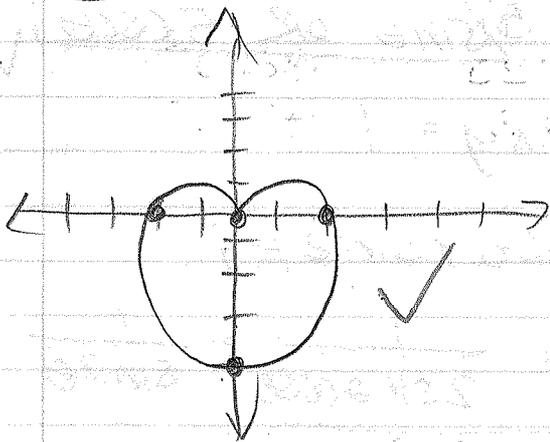
$$(36) r = -3 \cos \theta$$



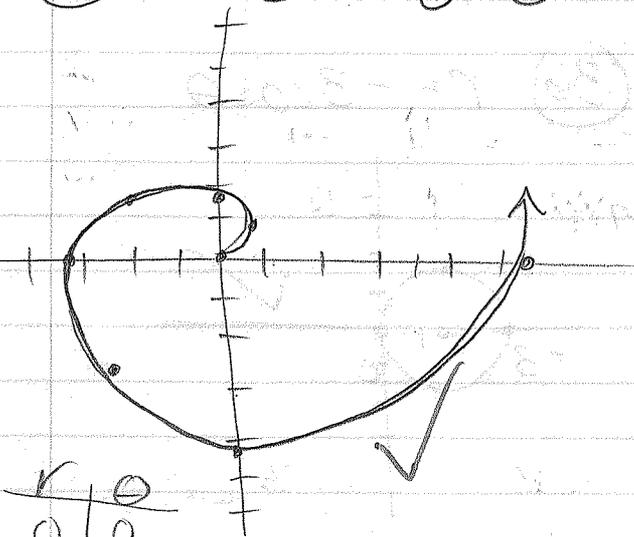
(38)  $r = 1 - 3\cos\theta$



(37)  $r = 2 - 2\sin\theta$

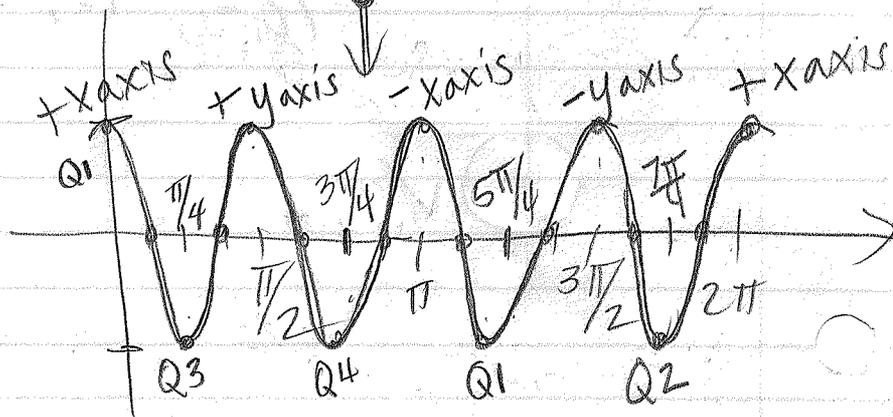
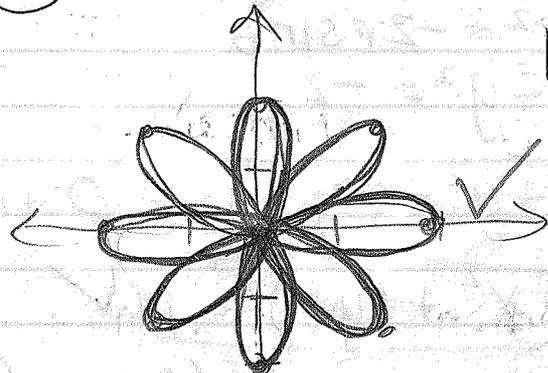


(39)  $r = \theta \quad \theta \geq 0$



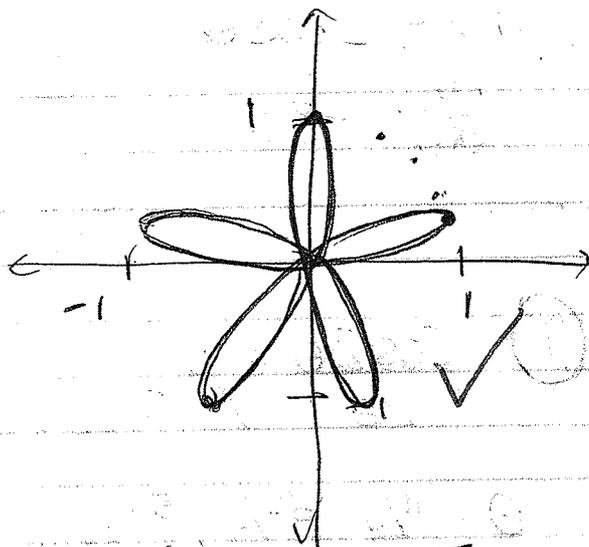
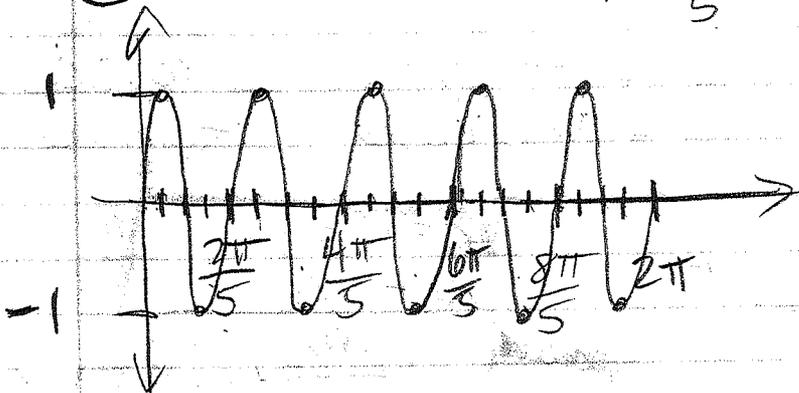
(45)  $r = 2\cos 4\theta$

$\rho = \frac{2\pi}{4} = \frac{\pi}{2}$



$r$	$\theta$
0	0
$\pi/4$	$\pi/4$
$\pi/2$	$\pi/2$
	$\vdots$
	$\vdots$

46)  $r = \sin 5\theta$       $\rho = \frac{2\pi}{5}$



$\theta = \frac{\pi}{10} \cdot \frac{180}{\pi} = 18^\circ$

56) a) VI  
c) IV  
e) II

b) III  
d) V  
f) I

59)  $r = \frac{1}{\theta}$       $\theta = \pi$

$\frac{dr}{d\theta} = -\theta^{-2} = -\frac{1}{\theta^2}$

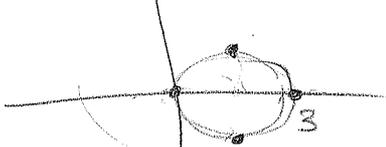
$\frac{dy}{dx} = \frac{\theta^2 \left( -\frac{1}{\theta^2} (\sin \theta) + \cos \theta \left( \frac{1}{\theta} \right) \right)}{\theta^2 \left( -\frac{1}{\theta^2} (\cos \theta) - \sin \theta \left( \frac{1}{\theta} \right) \right)} = \frac{-\sin \theta + \theta \cos \theta}{-\cos \theta - \theta \sin \theta}$

$\frac{dy}{dx} \Big|_{\theta=\pi} = \frac{-\sin \pi + \pi \cos \pi}{-\cos \pi - \pi \sin \pi} = \frac{0 + -\pi}{1 - 0} = -\pi$

61)  $r = 1 + \cos \theta$       $\theta = \pi/6$   
 $dr/d\theta = -\sin \theta$

$\frac{dy}{dx} = \frac{-\sin \theta (\sin \theta) + \cos \theta (1 + \cos \theta)}{-\sin \theta (\cos \theta) - \sin \theta (1 + \cos \theta)} = \frac{-\sin^2 \theta + \cos \theta + \cos^2 \theta}{-2\sin \theta \cos \theta - \sin \theta}$

$\frac{dy}{dx} = \frac{\cos \theta + \cos 2\theta}{-\sin \theta - \sin 2\theta}$       $\frac{dy}{dx} \Big|_{\theta=\pi/6} = \frac{\cos \pi/6 + \cos \pi/3}{-\sin \pi/6 - \sin \pi/3} = \frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{-\frac{1}{2} - \frac{\sqrt{3}}{2}} = -1$



(63)  $r = 3 \cos \theta$   
 $\frac{dr}{d\theta} = -3 \sin \theta$

$$\frac{dy}{dx} = \frac{-3 \sin \theta (\sin \theta) + \cos \theta (3 \cos \theta)}{-3 \sin \theta (\cos \theta) - \sin \theta (3 \cos \theta)}$$

$$\frac{dy}{dx} = \frac{\cos^2 \theta - \sin^2 \theta}{-2 \sin \theta \cos \theta} = \frac{\cos 2\theta}{-\sin 2\theta}$$

(4)  $\cos 2\theta = 0$   
 $2\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

polar

$r = 3 \cos \pi/4$   
 $r = 3\sqrt{2}/2$   
 $(3\sqrt{2}/2, \pi/4)$

$r = 3 \cos 3\pi/4$   
 $r = -3\sqrt{2}/2$   
 $(-3\sqrt{2}/2, 3\pi/4)$

$(\frac{-3\sqrt{2}}{2}, \frac{5\pi}{4})$

$x = r \cos \theta = 3 \cos^2 \theta$   
 $y = r \sin \theta = 3 \cos \theta \sin \theta$

$x(\pi/4) = 3(1/2) = 3/2$      $y(\pi/4) = 3/2$  rectangular same as  
 $* (3/2, 3/2)$     &     $(+3/2, -3/2)$

(V)  $-\sin 2\theta = 0$   
 $2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$   
 $\theta = 0, \pi/2, \pi, 3\pi/2, 2\pi$

Rectangular

$x(0) = 3$      $y(0) = 0$   
 $x(\pi/2) = 0$      $y(\pi/2) = 0$   
 $x(\pi) = 3$      $y(\pi) = 0$   
 $x(3\pi/2) = 0$      $y(3\pi/2) = 0$

$* (3, 0) \& (0, 0)$

polar

$r = 3 \cos 0$   
 $r = 3$   
 $(3, 0)$   
 $r, \theta$

$r = 3 \cos \pi/2$   
 $r = 0$   
 $(0, \pi/2)$

(71)  $0 \leq \theta \leq 4\pi$  ✓  
 $r = 1 + 2 \sin(\theta/2)$

(75)  $0 \leq \theta \leq 8\pi$  ✓

(73)  $0 \leq \theta \leq 2\pi$  ✓  
 $r = e^{\sin \theta} - 2 \cos(4\theta)$

$p = 2\pi \cdot \frac{4}{9} = \frac{8\pi}{9} \times 9$   
 $r = \sin(\frac{9\theta}{4})$   
 to get all petals

(67)  $r = \cos 2\theta$   
 $\frac{dr}{d\theta} = -2\sin 2\theta$

$\frac{dy}{dx} = \frac{-2\sin 2\theta \cdot \sin \theta + \cos \theta (\cos 2\theta)}{-2\sin 2\theta \cos \theta - \sin \theta (\cos 2\theta)}$

$\frac{dy}{dx} = \frac{-2 \cdot 2\sin \theta \cos \theta \sin \theta + \cos \theta (\cos^2 \theta - \sin^2 \theta)}{-2 \cdot 2\sin \theta \cos \theta \cos \theta - \sin \theta (\cos^2 \theta - \sin^2 \theta)}$

$\frac{dy}{dx} = \frac{-4\sin^2 \theta \cos \theta + \cos^3 \theta - \sin^2 \theta \cos \theta}{-4\cos^2 \theta \sin \theta - \sin \theta \cos^2 \theta + \sin^3 \theta}$

$\frac{dy}{dx} = \frac{\cos \theta (-5\sin^2 \theta + \cos^2 \theta)}{\sin \theta (-5\cos^2 \theta + \sin^2 \theta)} = \frac{\cos \theta (-5\sin^2 \theta + 1 - \sin^2 \theta)}{\sin \theta (-5\cos^2 \theta + 1 - \cos^2 \theta)}$

$\frac{dy}{dx} = \frac{\cos \theta (1 - 6\sin^2 \theta)}{\sin \theta (1 - 6\cos^2 \theta)}$  (H)  $\cos \theta = 0$   $1 - 6\sin^2 \theta = 0$   
 $\theta = \pi/2, 3\pi/2, \dots$   $1 = 6\sin^2 \theta$   
 $\sqrt{\quad}$   $1/6 = \sin^2 \theta$

(V)  $\sin \theta = 0$   
 $\theta = 0, \pi, 2\pi$  ✓

$\pm \frac{1}{\sqrt{6}} = \sin \theta$

$\theta = \arcsin(\pm 1/\sqrt{6})$  ✓

$\cos^2 \theta = 1/6$   
 $\theta = \arccos(\pm 1/\sqrt{6})$  ✓

$x = r \cos \theta$

$y = r \sin \theta$

horizontal

Vertical

$(1, 0)$   
 $(1, \pi)$

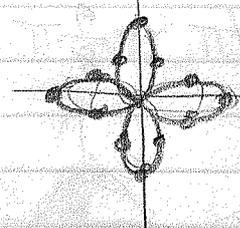
$(-2/3, \arccos(\pm 1/\sqrt{6}))$   
 $(-2/3, -\arccos(\pm 1/\sqrt{6}))$   
 $(2/3, \arccos(\pm 1/\sqrt{6}))$  ✓  
 $(2/3, -\arccos(1/\sqrt{6}))$

$(-1, \pi/2)$   
 $(-1, 3\pi/2)$

$(2/3, \arcsin(1/\sqrt{6}))$   
 $(2/3, \arcsin(-1/\sqrt{6}))$   
 $(-2/3, \arcsin(1/\sqrt{6}))$   
 $(-2/3, -\arcsin(1/\sqrt{6}))$

polar ↗

polar ↗



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①  $r = \sqrt{\theta} \quad 0 \leq \theta \leq \pi/4$

$$A = \frac{1}{2} \int_0^{\pi/4} (\sqrt{\theta})^2 d\theta = \frac{1}{2} \int_0^{\pi/4} \theta d\theta = \left[ \frac{1}{4} \theta^2 \right]_0^{\pi/4} =$$

$$\boxed{\frac{\pi^2}{64}} \checkmark$$

⑤  $A = \frac{1}{2} \int_0^{\pi} \theta^2 d\theta = \left[ \frac{1}{6} \theta^3 \right]_0^{\pi} = \frac{1}{6} \pi^3 - 0 = \frac{\pi^3}{6} \checkmark$

⑦  $A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} (4 + 3 \sin \theta)^2 d\theta =$

$$\frac{1}{2} \int_{-\pi/2}^{\pi/2} 16 + 24 \sin \theta + 9 \sin^2 \theta d\theta =$$

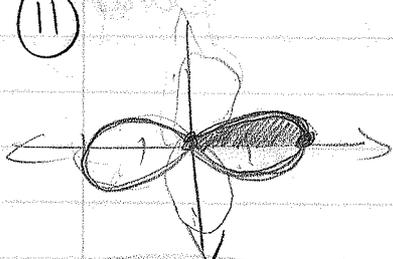
$$\frac{1}{2} \int_{-\pi/2}^{\pi/2} 16 + 24 \sin \theta + \frac{9}{2} (1 - \cos 2\theta) d\theta =$$

$$\frac{1}{2} \left[ 16\theta - 24 \cos \theta + \frac{9}{2} \theta - \frac{9}{2} \cdot \frac{1}{2} \sin 2\theta \right]_{-\pi/2}^{\pi/2}$$

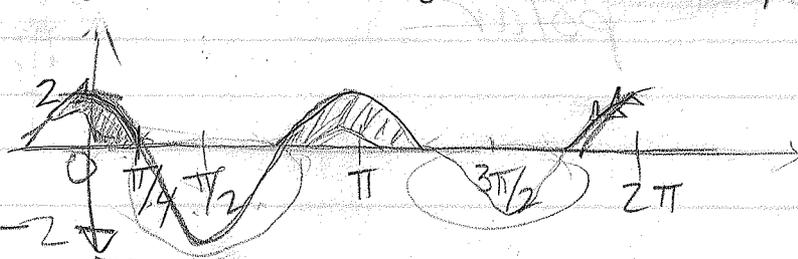
$$\left[ 4\pi - \cancel{12 \cos \frac{\pi}{2}} + \frac{9}{8} \pi - \cancel{\frac{9}{8} \sin \pi} \right] - \left[ -4\pi - \cancel{12 \cos \left( \frac{-\pi}{2} \right)} - \frac{9}{8} \pi - \cancel{\frac{9}{8} \sin \left( \frac{-\pi}{2} \right)} \right]$$

$$4\pi + \frac{9}{8} \pi + 4\pi + \frac{9}{8} \pi = 8\pi + \frac{9}{4} \pi = \frac{41}{4} \pi \checkmark$$

⑪



$$P = \frac{2\pi}{2} = \pi$$



$$r = \pm \sqrt{4 \cos 2\theta}$$

$$r = 2 \cos 2\theta$$

$$r = \sqrt{4 \cos 2\theta}$$

$$r = 2 \sqrt{\cos 2\theta}$$

$$\textcircled{11} \quad 4 \cdot \frac{1}{2} \int_0^{\pi/4} (4 \cos 2\theta) d\theta = 2 \int_0^{\pi/4} 4 \cos 2\theta d\theta$$

$$2 [2 \sin 2\theta]_0^{\pi/4} = 4 [\sin \pi/2] - 4 [\sin 0] = \textcircled{4} \checkmark$$

$$\textcircled{15} \quad 4 \cdot \frac{1}{2} \int_0^{\pi/2} (2 + \cos 6\theta)^2 d\theta = 4 \cdot \frac{1}{2} \int_0^{\pi/2} 4 + 4 \cos 6\theta + \cos^2 6\theta d\theta$$

$$4 \cdot \frac{1}{2} \int_0^{\pi/2} 4 + 4 \cos 6\theta + \frac{1}{2} (1 + \cos 12\theta) d\theta =$$

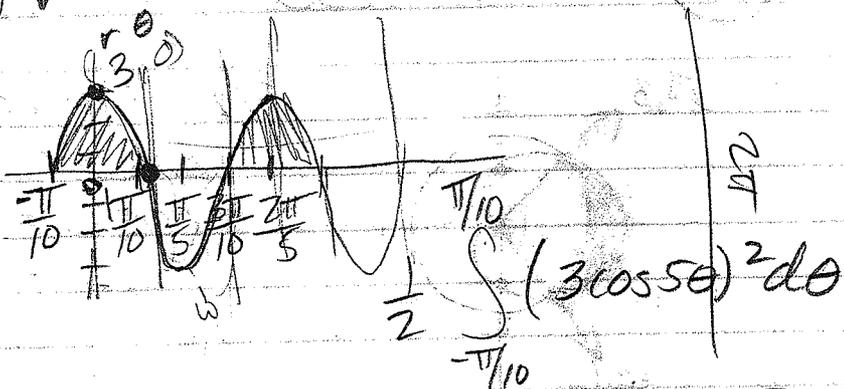
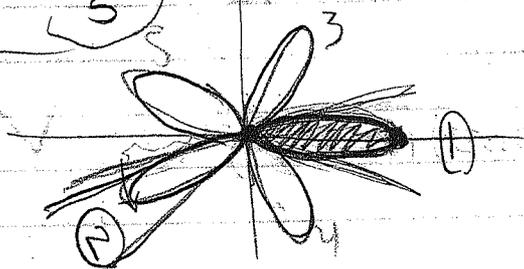
$$\frac{1}{2} \cdot 4 [4\theta + \frac{4}{6} \sin 6\theta + \frac{1}{2}\theta + \frac{1}{2} \cdot \frac{1}{2} \sin 12\theta]_0^{\pi/2}$$

$$[16(\pi/2) + \frac{8}{3} \sin 3\pi + 2(\pi/2) + \sin 6\pi] - 0$$

$$8\pi + \pi = \textcircled{\frac{9\pi}{2}} \checkmark$$

$$\textcircled{19} \quad r = 3 \cos 5\theta$$

$$p = \frac{2\pi}{5}$$



$$\frac{9}{2} \int_{-\pi/10}^{\pi/10} \cos^2 5\theta d\theta = \textcircled{\frac{9\pi}{20}} \checkmark$$

$$0 = 3 \cos 5\theta$$

$$0 = \cos 5\theta$$

$$5\theta = \pi/2, 3\pi/2, \dots$$

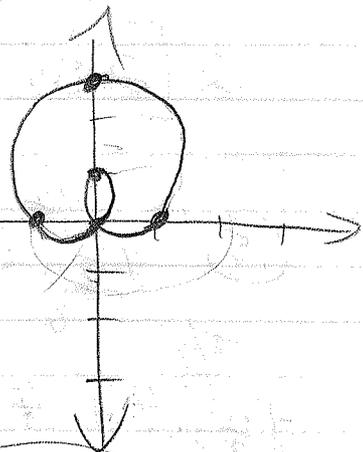
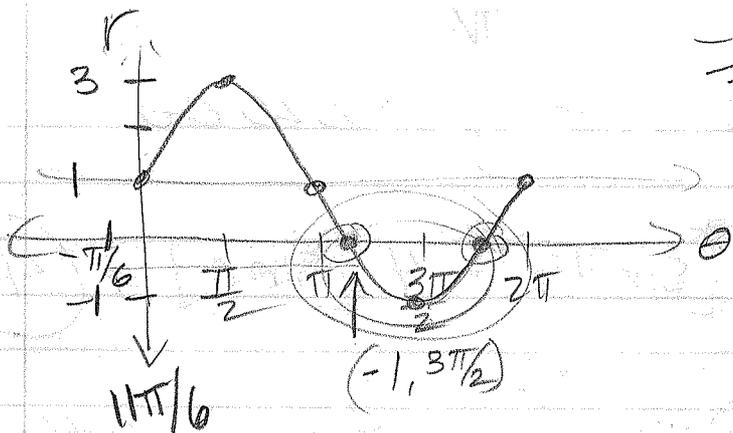
$$\theta = \pi/10, 3\pi/10, \dots$$

(21)  $r = 1 + 2\sin\theta$

$0 = 1 + 2\sin\theta$

$-\frac{1}{2} = \sin\theta$

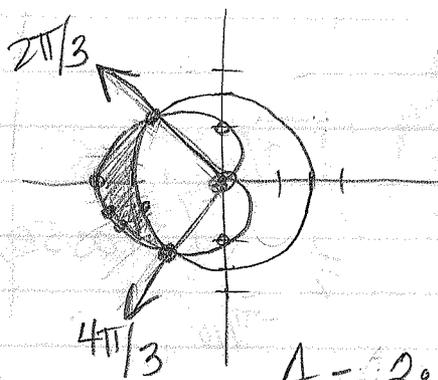
$\theta = \frac{7\pi}{6}$  OR  $\frac{11\pi}{6}$



$\frac{1}{2} \int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)^2 d\theta = \pi - \frac{3\sqrt{3}}{2}$

$L = \int_a^b \sqrt{(f'(x))^2 + 1^2} dx$   
 $x = r\cos\theta$   
 $y = r\sin\theta$

(23)  $r = 1 - \cos\theta$      $r = \frac{3}{2}$



$\frac{3}{2} = 1 - \cos\theta$

$\frac{1}{2} = -\cos\theta$

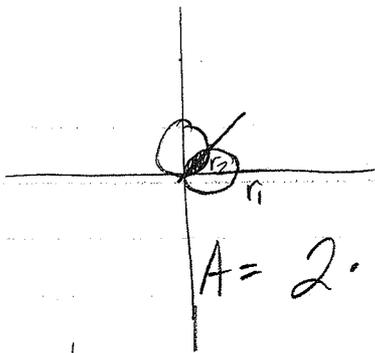
$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$  &  $\frac{4\pi}{3}$

$A = 2 \cdot \frac{1}{2} \int_{2\pi/3}^{4\pi/3} (1 - \cos\theta)^2 - \left(\frac{3}{2}\right)^2 d\theta$

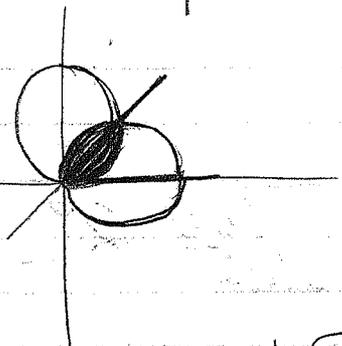
$\frac{9\sqrt{3}}{8} - \frac{\pi}{4}$

(29)  $r_2 = \sin \theta$  &  $r_1 = \cos \theta$



$\sin \theta = \cos \theta$   
 $\theta = \pi/4$

$A = 2 \cdot \frac{1}{2} \int_0^{\pi/4} (\sin \theta)^2 d\theta = \int_0^{\pi/4} \sin^2 \theta d\theta$



$\frac{1}{2} \int_0^{\pi/4} (1 - \cos 2\theta) d\theta$

$\frac{1}{2} \left[ \theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi/4}$

$\frac{1}{2} \left[ \frac{\pi}{4} - \frac{1}{2} \sin \frac{\pi}{2} \right] - \frac{1}{2} [0 - 0]$

$\frac{\pi}{8} - \frac{1}{4} \checkmark$

(45)  $r = 5 \cos \theta$       $\frac{dr}{d\theta} = -5 \sin \theta$

$L = \int_0^{3\pi/4} \sqrt{(5 \cos \theta)^2 + (-5 \sin \theta)^2} d\theta = \int_0^{3\pi/4} \sqrt{25(\cos^2 \theta + \sin^2 \theta)} d\theta$

$\int_0^{3\pi/4} 5 d\theta = [5\theta]_0^{3\pi/4} = \frac{15\pi}{4} \checkmark$

$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$

$L = \int_{\theta=a}^{\theta=b} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + (r)^2} d\theta$

$\frac{(r' \sin \theta + r \cos \theta)^2}{(r' \cos \theta - r \sin \theta)^2} + 1$

$\sqrt{\frac{(r' \sin \theta + r \cos \theta)^2 + (r' \cos \theta - r \sin \theta)^2}{(r' \cos \theta - r \sin \theta)^2}}$

$\frac{dr^2 \sin^2 \theta}{d\theta} + 2 \frac{dr}{d\theta} \cdot r \sin \theta \cos \theta + r^2 \cos^2 \theta + \frac{dr^2 \cos^2 \theta}{d\theta} - 2 \frac{dr}{d\theta} r \sin \theta \cos \theta + r^2 \sin^2 \theta$

(47)  $r = 2^\theta$   $0 \leq \theta \leq 2\pi$

$$\frac{dr}{d\theta} = \ln 2 \cdot 2^\theta$$

$$L = \int_0^{2\pi} \sqrt{(2^\theta)^2 + (\ln 2 \cdot 2^\theta)^2} d\theta =$$

$$\int_0^{2\pi} \sqrt{2^{2\theta} + (\ln 2)^2 \cdot 2^{2\theta}} d\theta$$

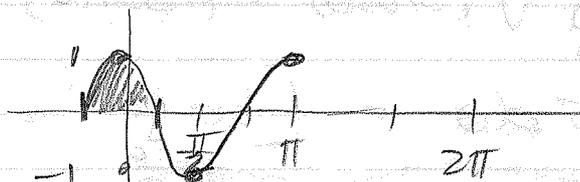
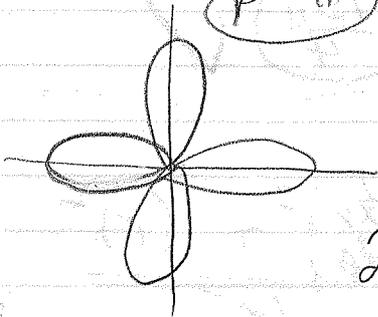
$$\int_0^{2\pi} 2^\theta \sqrt{1 + (\ln 2)^2} d\theta = \left[ \frac{\sqrt{1 + (\ln 2)^2} 2^\theta}{\ln 2} \right]_0^{2\pi} =$$

$$\frac{\sqrt{1 + \ln^2 2} \cdot 2^{2\pi}}{\ln 2} - \frac{\sqrt{1 + \ln^2 2}}{\ln 2}$$

$$\frac{(2^{2\pi} - 1) \sqrt{1 + \ln^2 2}}{\ln 2} \checkmark$$

(51)  $r = \cos 2\theta$

$\rho = \pi$



Area

$$2 \cdot \frac{1}{2} \int_0^{\pi/4} \cos^2 2\theta d\theta = \frac{1}{2} \int_0^{\pi/4} 1 + \cos 4\theta d\theta$$

$$\frac{1}{2} \left[ \theta + \frac{1}{4} \sin 4\theta \right]_0^{\pi/4} =$$

$$\frac{1}{2} \left[ \frac{\pi}{4} + \frac{1}{4} \sin \pi \right] - 0 = \frac{\pi}{8} \checkmark$$

$$(53) \quad L = \int_0^{4\pi} \sqrt{\cos^8 \frac{\theta}{4} + \cos^6 \frac{\theta}{4} \sin^2 \frac{\theta}{4}} d\theta = \left(\frac{16}{3}\right) \checkmark$$

$$r = (\cos \theta/4)^4$$

$$\frac{dr}{d\theta} = 4(\cos \frac{\theta}{4})^3 \cdot -\sin \frac{\theta}{4} \cdot \frac{1}{4}$$

$$\frac{dr}{d\theta} = -\cos^3 \frac{\theta}{4} \sin \frac{\theta}{4}$$

(51) Length

$$L = \int_{-\pi/4}^{\pi/4} \sqrt{\cos^2 2\theta + (2\sin 2\theta)^2} d\theta$$

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$$r = \cos 2\theta$$

$$\frac{dr}{d\theta} = -2\sin 2\theta$$

$$(7) \quad \frac{1}{4} \left[ 16\theta - 24\cos\theta + \frac{9}{2}\theta - \frac{9}{4}\sin 2\theta \right]_0^{2\pi}$$

$$\frac{1}{4} [32\pi - 24 + 9\pi - 0] - \frac{1}{4} [-24]$$

$$8\pi - 6 + \frac{9\pi}{4} + 6 = \frac{32\pi}{4} + \frac{9\pi}{4} = \left(\frac{41\pi}{4}\right) \text{ Same!}$$